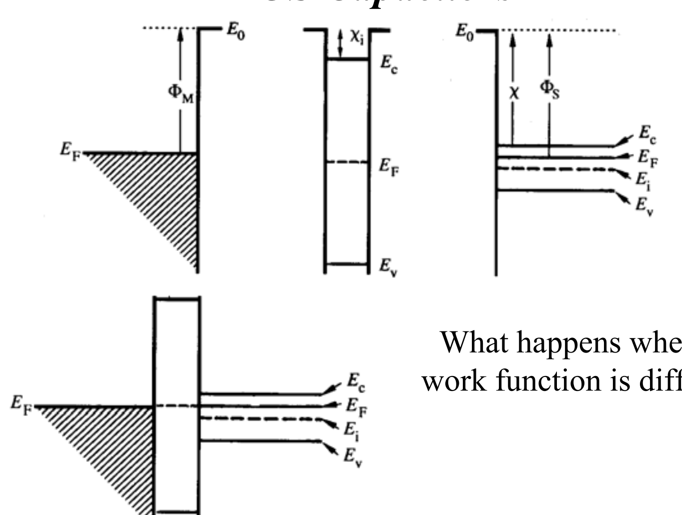
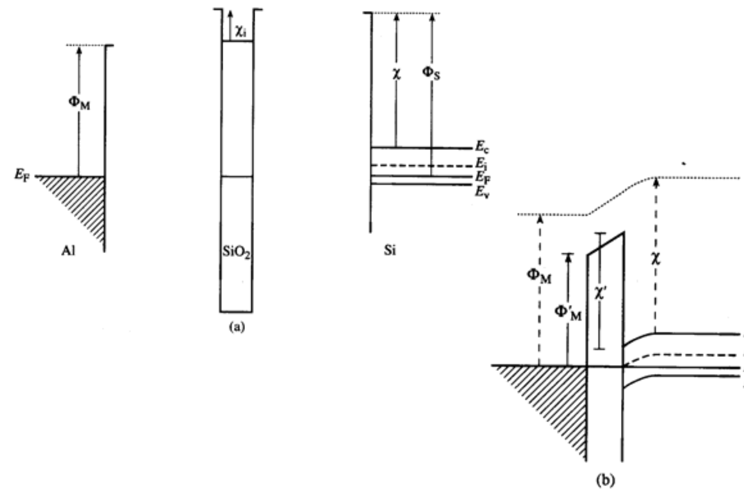
A quick review of MOS Capacitors

MOS Capacitors

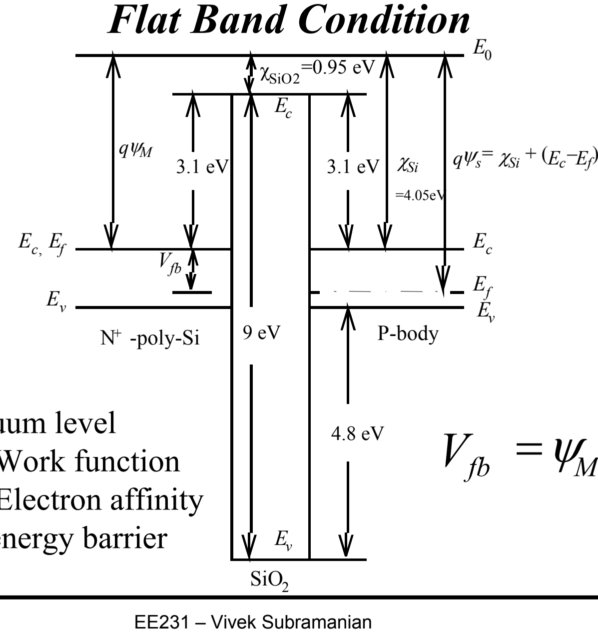


What happens when the work function is different?

MOS Band Diagram – Different Work function



Flat Band Condition



E_0 : Vacuum level
$E_0 - E_f$: Work function
$E_0 - E_c$: Electron affinity
Si/SiO_2 energy barrier

$$V_{fb} = \psi_M - \psi_s$$

Non-Flat-band conditions

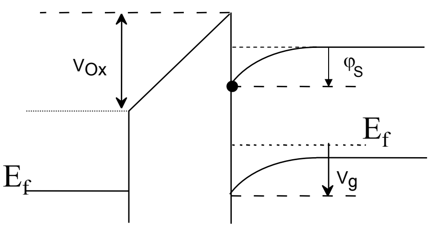
What if

$V_g \neq V_{fb}$

$V_g = V_{fb} + V_{Ox} + \varphi_S$

voltage across the substrate, i.e. the band bending in the substrate, also called surface potential

voltage across the oxide

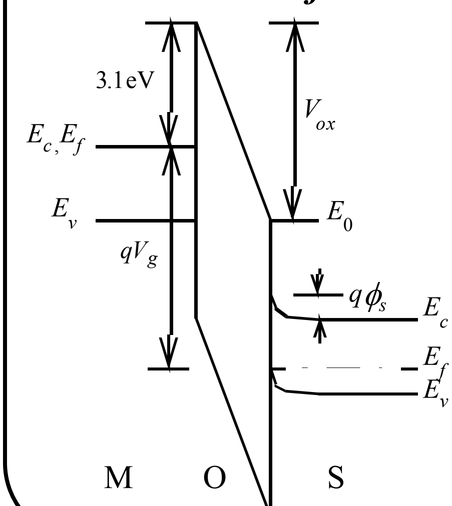


substrate charge

$$V_{Ox} = \frac{-Q_S}{C_{Ox}}$$

$$Q_S = Q_{dep} + Q_{inv} + Q_{acc}$$

Surface Accumulation



$$V_g = V_{fb} + \phi_s + V_{ox}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

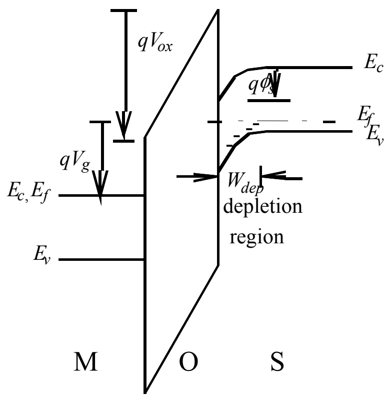
ϕ_s is negligible

$$V_{Ox} = \frac{-Q_S}{C_{Ox}}$$

$$V_g = V_{fb} + V_{Ox} + \varphi_S$$

$$Q_S = -C_{Ox} V_{Ox}$$
$$= -C_{Ox}\left(V_g - V_{fb}\right)$$

Depletion



$$V_g = V_{fb} + V_{Ox} + \varphi_S$$

$$V_{Ox} = \frac{-Q_S}{C_{Ox}} = \frac{-Q_{dep}}{C_{Ox}}$$

$$= \frac{+qN_a X_{dep}}{C_{Ox}}$$

$$= \frac{\sqrt{2qN_a \varepsilon_S \varphi_S}}{C_{Ox}}$$

Can solve for $\varphi_S(V_g)$ and $V_{Ox}(V_g)$

Threshold (of Inversion)

$$n_S = p_0 = N_a$$

$(E_C - E_f)_S = (E_f - E_V)_{bulk}$ $\quad C = D$

At Vt $\varphi_S = 2\ \varphi_B \approx 0.8V$ $\quad A = B$

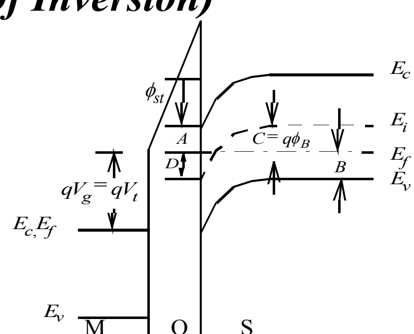
$$\varphi_B = \frac{1}{q}\left(E_i - E_f\right)_{bulk}$$

$$= \frac{kT}{q}\ln\frac{N_a}{n_i} \approx 0.4V$$

(Alternative definition: $\varphi_{S,th} = \varphi_B + 0.45V$)

$$V_T = V_g\left(\varphi_S = 2\varphi_B\right) = V_{fb} + \varphi_S + V_{Ox}$$

$$V_T = \underbrace{V_{fb} + 2\ \varphi_B}_{0.15V} + \frac{\sqrt{2q\varepsilon_S N_a 2\varphi_B}}{C_{Ox}}$$



$$n = N_C e^{-\left(E_C - E_f\right)/kT}$$

$$p = N_V e^{-\left(E_f - E_V\right)/kT}$$

$$\approx 10^{19}\ cm^{-3}$$

Inversion

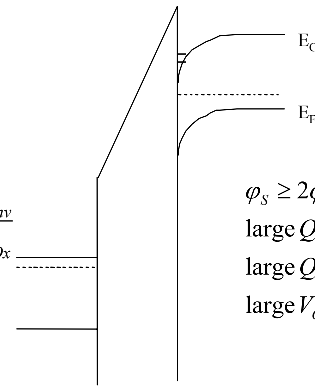
$$\varphi_S \geq 2\varphi_B \quad \varphi_S \sim 2\varphi_B$$

$$V_g = V_{fb} + 2\varphi_B - \frac{Q_{dep} + Q_{inv}}{C_{Ox}}$$

$$= V_{fb} + 2\varphi_B + \frac{\sqrt{2q\varepsilon_S N_a 2\varphi_B}}{C_{Ox}} - \frac{Q_{inv}}{C_{Ox}} = V_T - \frac{Q_{inv}}{C_{Ox}}$$

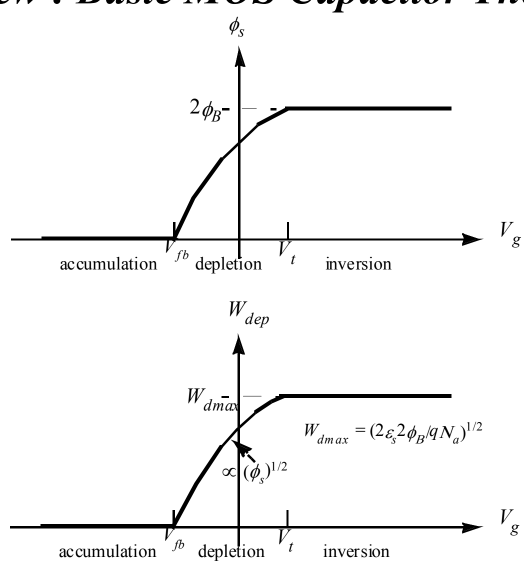
$$V_T = V_{fb} + 2\varphi_B - \frac{Q_{dep}}{C_{Ox}}$$

$$Q_{inv} = -C_{Ox}\left(V_g - V_T\right)$$

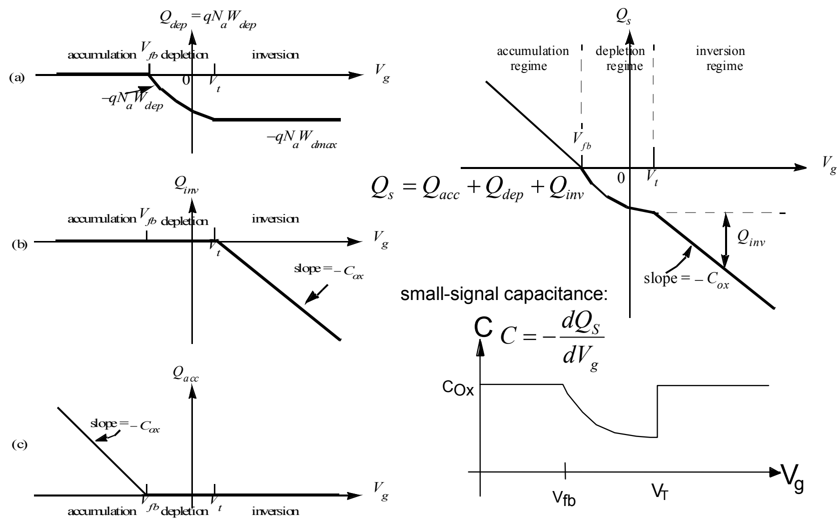


$\varphi_S \geq 2\varphi_B$
large Q_{inv}
large Q_S
large V_{Ox}

Review : Basic MOS Capacitor Theory



Review : Basic MOS Capacitor Theory



Advanced MOSCAP Physics and Technology

So were we too simplistic?

- Problem with previous analysis
  - Assumes that there are no free carriers in the depletion region (depletion approximation)
  - Obviously, this is not true (else, how could we have inversion charge?)

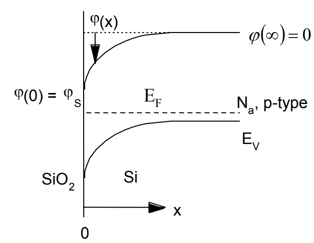
$$p(x) = p_0 e^{-q\varphi(x)/kT} = p_0 e^{-\beta\varphi(x)}$$

(p_0 ← N_a)

$$\beta \equiv \frac{q}{kT}$$

$$n(x) = n_0 e^{q\varphi(x)/kT} \quad \text{Eq. 3-9}$$

(n_0 ← $\frac{n_i^2}{N_a}$)



A more general and accurate MOSCAP analysis

- If we don't assume complete absence of carriers in the depletion region, we have:

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho(x)}{\varepsilon_S} = -\frac{q}{\varepsilon_S}\left(p + N_d^+ - n - N_a^-\right)$$

$$= -\frac{q}{\varepsilon_S}\left(p_0 e^{-\beta\varphi(x)} - n_0 e^{\beta\varphi(x)} - N_a + N_d\right)$$

- Now, we can proceed by noting that:

$$\frac{d^2\varphi}{dx^2} = \frac{1}{2}\frac{d}{d\varphi}\left(\frac{d\varphi}{dx}\right)^2$$

Calculation of E(x)

- Therefore, we have:

$$\frac{d^2\varphi}{dx^2} = \frac{1}{2}\frac{d}{d\varphi}\left(\frac{d\varphi}{dx}\right)^2 = \frac{-q}{\varepsilon_S}\left(p_0 e^{-\beta\varphi} - n_0 e^{\beta\varphi} - N_a + N_d\right)$$

$$\Rightarrow \left(\frac{d\varphi}{dx}\right)^2 = \frac{-2q}{\varepsilon_S}\int_0^{\varphi}\left(p_0 e^{-\beta\varphi} - n_0 e^{\beta\varphi} - N_a + N_d\right)d\varphi$$

$$= \frac{-2q}{\varepsilon_S \beta}\left[-p_0(e^{-\beta\varphi} + \beta\varphi - 1) - n_0(e^{\beta\varphi} - \beta\varphi - 1)\right]$$

$$\frac{d\varphi}{dx} = \left[\frac{2q}{\varepsilon_S \beta}\left(p_0(e^{-\beta\varphi} + \beta\varphi - 1) + n_0(e^{\beta\varphi} - \beta\varphi - 1)\right)\right]^{1/2}$$

- Which is –E(x)

Calculation of Q_S

- Now, we can solve this equation at x=0 to find the peak field. By Gauss' law, we can therefore find the total charge in the silicon

$$Q_S = -\varepsilon_S E_S \qquad \text{by Gauss's Law}$$

$$= \varepsilon_S \left.\frac{d\varphi}{dx}\right|_S = \varepsilon_S \left.\frac{d\varphi}{dx}\right|_{\varphi=\varphi_S}$$

$$= \sqrt{\frac{2\varepsilon_S q}{\beta}}\left[p_0\left(e^{-\beta\varphi_S} + \beta\varphi_S - 1\right) + n_o\left(e^{\beta\varphi_S} - \beta\varphi_S - 1\right)\right]^{1/2}$$

- This equation is valid in all regions of operation, since we haven't made any region-specific assumptions so far

Calculation of inversion and depletion charge

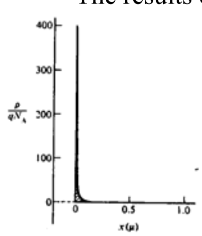
- From the previous analysis, we can also calculate the various charge components, namely the inversion layer charge and the depletion layer charge (and, of course, the accumulation charge in accumulation).
- For example, in inversion, we find:

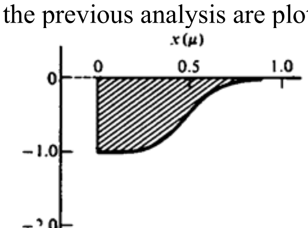
$$Q_I = -q\int_0^{x_c} n(x)dx = -q\int_{\varphi_{surf}}^{\varphi_c} \frac{n(\varphi)d\varphi}{d\varphi / dx}$$

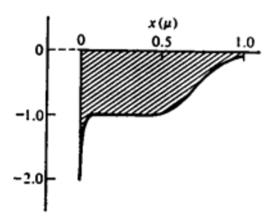
- Since we know E(x) and n(x), we can solve for the total inversion layer charge.
- Similarly, we can also solve for Q_B, which consists of ionized acceptors and holes.

Graphical analysis of charge distributions

- The results of the previous analysis are plotted below:







Accumulation Depletion Onset of Inversion

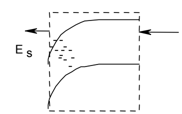
- Specific conclusions:
  1. The inversion layer charge is extremely close to the surface
  2. There are very few mobile carriers in the depletion region

i.e., our simplistic analysis in the previous section is actually reasonably accurate.

Extraction of C-V characteristics

- From our equation for Q_S, we can also determine more accurate C-V characteristics that predicted by the previous analysis.

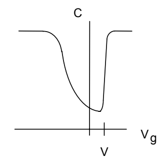
$$Q_S = \sqrt{\frac{2\varepsilon_S q}{\beta}}\left[p_0\left(e^{-\beta\varphi_S} + \beta\varphi_S - 1\right) + n_o\left(e^{\beta\varphi_S} - \beta\varphi_S - 1\right)\right]^{1/2}$$



$$C_S = -\frac{dQ_S}{d\varphi_S} = \sqrt{\frac{q\varepsilon_S\beta}{2}}\frac{\left|p_0\left(-e^{-\beta\varphi_S} + 1\right) + n_o\left(e^{\beta\varphi_S} - 1\right)\right|}{\left[p_0\left(e^{-\beta\varphi_S} + \beta\varphi_S - 1\right) + n_o\left(e^{\beta\varphi_S} - \beta\varphi_S - 1\right)\right]^{1/2}}$$

$$= \sqrt{\frac{q\varepsilon_S\beta}{2}}\frac{\left|p_S - n_S - N_a + N_d\right|}{\sqrt{p_S + n_S + (\beta\varphi_S - 1)(N_a - N_d) - 2n_0}}$$

$$Q_S = Q_{inv} + Q_{dep} + Q_{acc}$$



Extraction of C at V_T

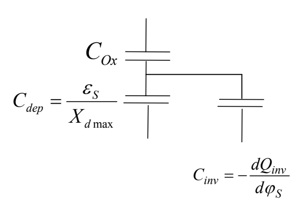
- We can use this equation to determine specific capacitance values

$$C_S = \sqrt{\frac{q\varepsilon_S\beta}{2}}\frac{\left|p_S - n_S - N_a + N_d\right|}{\sqrt{p_S + n_S + (\beta\varphi_S - 1)(N_a - N_d) - 2n_0}}$$

- At $V_G = V_T$, we have:

$$C_S = \sqrt{\frac{q\beta\varepsilon_S}{2}}\frac{2N_a}{\sqrt{2\beta\varphi_B N_a}} = \sqrt{\frac{q\varepsilon_S N_a}{\varphi_B}} = \frac{\varepsilon_S}{\sqrt{\varphi_B\varepsilon_S / qN_a}} = \frac{\varepsilon_S}{X_{d\max}/2}$$

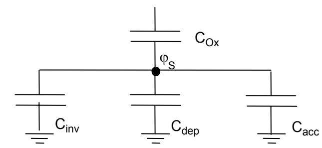
- Where $X_{d\max} = \sqrt{\frac{2\varepsilon_S 2\varphi_B}{qN_a}}$



- This value is different from the value predicted by the simple model by a factor if 2, since the latter does not include the inversion charge present at V_T

Extraction of C at V_{FB}

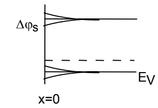
- Similarly, at $V_G = V_{FB}$, we find that:



$$C_S(V_{fb}) = \frac{\varepsilon_S}{\underbrace{\sqrt{\frac{\varepsilon_S}{qN_a\beta}}}_{\text{X}_{\text{dep}}\text{ for band bending of kT/2q}}} \Big\} \text{ Debye length}$$

X_{dep} for band bending of kT/2q

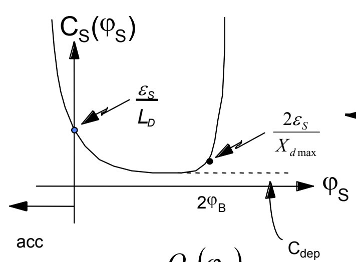
$$\frac{1}{\beta} = \frac{kT}{q}$$



- Again, we find that the simplistic model is somewhat inaccurate, since it doesn't include the "wiggle" around flat-band, which affects the charge in the silicon. This effect is small, of course.

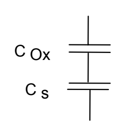
MOSCAP LFCV Characteristics

- We can plot the variation in C_S in the various regions to find the MOSCAP LFCV characteristics:



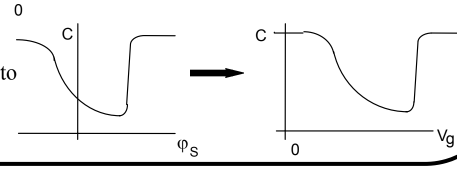
$$C_{dep} \approx \frac{\varepsilon_S}{X_{d\max}}$$

$$C_{inv} = \frac{\varepsilon_S}{X_{d\max}}$$



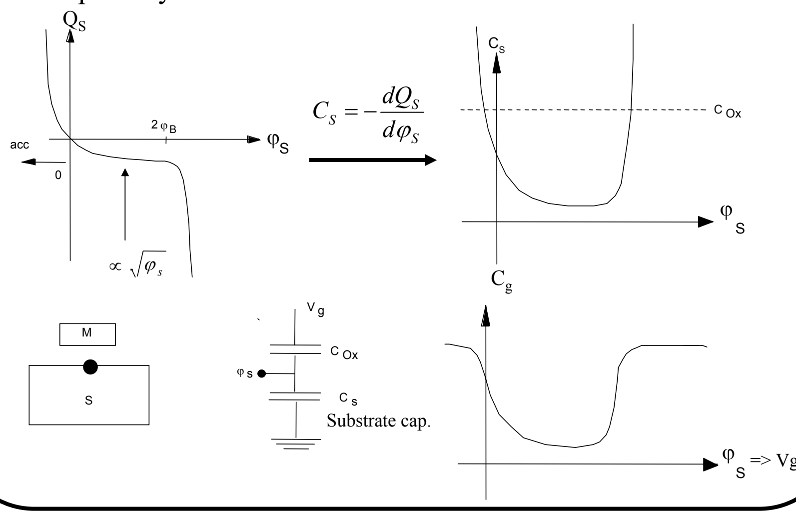
$$V_g = V_{fb} + \varphi_S - \frac{Q_S(\varphi_S)}{C_{Ox}}$$

LF : Q_{inv}, Q_{acc}, Q_{dep}, i.e., n, p respond to changes in φ_S, eg., $p = p_0 e^{-\beta\varphi}$, i.e. n and p are at equilibrium.



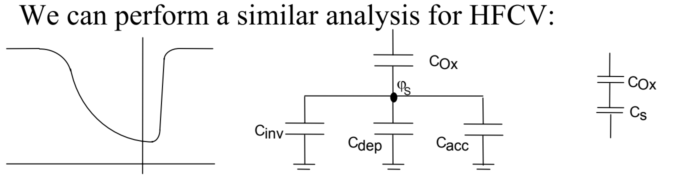
MOSCAP LFCV Characteristics

- Graphically:



MOSCAP HFCV Characteristics

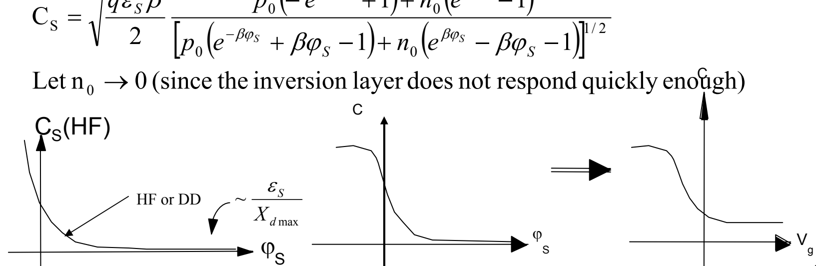
- We can perform a similar analysis for HFCV:



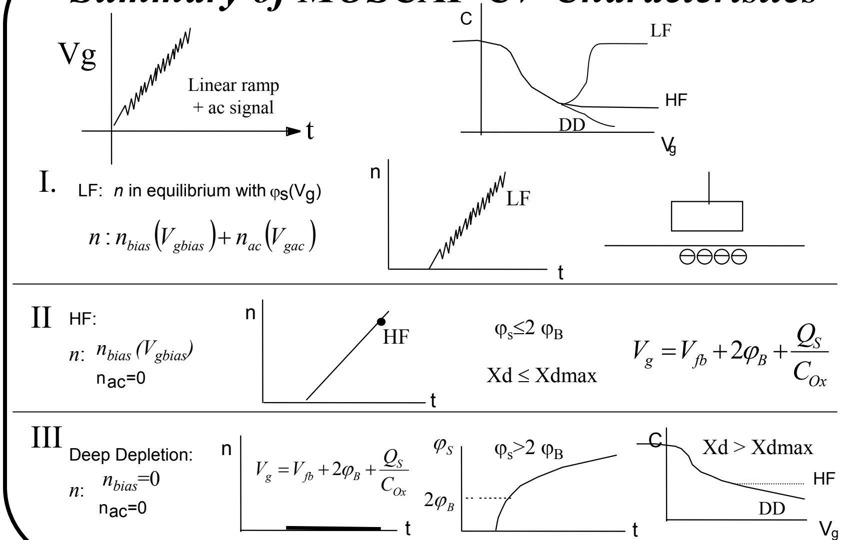
$C_S(\varphi_S)$ for HF?

$$C_S = \sqrt{\frac{q\varepsilon_S\beta}{2}}\frac{p_0\left(-e^{-\beta\varphi_S}+1\right)+n_0\left(e^{\beta\varphi_S}-1\right)}{\left[p_0\left(e^{-\beta\varphi_S}+\beta\varphi_S-1\right)+n_0\left(e^{\beta\varphi_S}-\beta\varphi_S-1\right)\right]^{1/2}}$$

Let $n_0 \to 0$ (since the inversion layer does not respond quickly enough)



Summary of MOSCAP CV Characteristics



The Charge Sheet Model

- Problems with the simple model
  - Inaccurate in depletion
  - Inaccurate in accumulation
  - Inaccurate in weak inversion ($2\varphi_B > \varphi_S > \varphi_B$)
- Problems with the general model
  - Requires numerical solution for Q_I

The charge sheet model provides a reasonable tradeoff between the two. It isn't as accurate in depletion or accumulation, but these regions aren't as important for MOSFET operation

Main Assumptions

- Mobile charge exists beyond the onset of weak inversion (i.e., $Q_I > 0$ for $\varphi_S > \varphi_B$
  - unlike the simple model, which assumes that the inversion charge is zero for $\varphi_S < 2\varphi_B$
- Mobile charge is present in a negligibly thin layer
  - similar to the simple model
- Depletion region has a sharp boundary
  - similar to the simple model
- The surface potential is not clamped past threshold
  - unlike the simple model, which assumes that φ_S is clamped at $2\varphi_B$ for all values of V_G past threshold

Derivation of Q_S

- As in the general model, we have:

$$\frac{d^2\varphi}{dx^2} = \frac{-q}{\varepsilon_S}\left(p_0 e^{-\beta\varphi} - n_0 e^{\beta\varphi} - N_a + N_d\right)$$

$$\frac{d\varphi}{dx} = \left[\frac{2q}{\varepsilon_S \beta}\left(p_0(e^{-\beta\varphi} + \beta\varphi - 1) + n_0(e^{\beta\varphi} - \beta\varphi - 1)\right)\right]^{1/2}$$

$$Q_S = \sqrt{\frac{2\varepsilon_S q}{\beta}}\left[p_0\left(e^{-\beta\varphi_S} + \beta\varphi_S - 1\right) + n_o\left(e^{\beta\varphi_S} - \beta\varphi_S - 1\right)\right]^{1/2}$$

- To simplify, assume we only care about Q_S in weak and strong inversion (i.e., $\varphi_S > \varphi_B$). Then:

$$|Q_S| = \sqrt{\frac{2qN_a\varepsilon_S}{\beta}\left(\beta\varphi_S + \frac{n_i^2}{N_a^2}e^{\beta\varphi_S}\right)} = |Q_{inv} + Q_{dep}|$$

Derivation of Q_{dep} and Q_{inv}

- Then, we can determine Q_{dep} as in the simple model, and subtract to find Q_{inv}

$$Q_{dep} = \sqrt{2qN_a\varepsilon_S\varphi_S}$$

$$Q_{inv} = \sqrt{\frac{2qN_a\varepsilon_S}{\beta}\left(\beta\varphi_S + \frac{n_i^2}{N_a^2}e^{\beta\varphi_S}\right)} - \sqrt{2qN_a\varepsilon_S\varphi_S}$$

$$V_g = V_{fb} + \varphi_S - \frac{Q_S(\varphi_S)}{C_{Ox}}$$

- Note that $Q_{inv}(V_G)$ still requires an iterative solution
- In strong inversion, since the exponent dominates, we can simplify to:

$$Q_{inv} = \sqrt{\frac{2q\varepsilon_S n_i^2 e^{\beta\varphi_S}}{\beta N_a}}$$

- This exponential dependence implies that large changes in Q_{inv} result from small changes in φ_S, which means that φ_S is essentially clamped, as was assumed in the simple model.

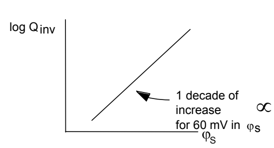
Simplification under weak inversion

- In weak inversion, $Q_{inv} < Q_B$. This allows us to simplify Q_{inv}:

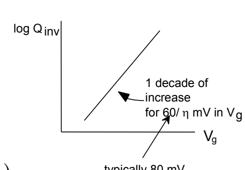
$$2\varphi_B > \varphi_S > \varphi_B$$

$$Q_{inv} = \sqrt{2\varepsilon_S qN_a\varphi_S}\left(\sqrt{1+\frac{n_i^2}{\varphi_S\beta N_a^2}e^{\beta\varphi_s}} - 1\right) \approx -\sqrt{\frac{q\varepsilon_S N_a}{2\varphi_S}}\frac{n_i^2}{\beta N_a^2}e^{\beta\varphi_s}$$

From first 2 terms of taylor series



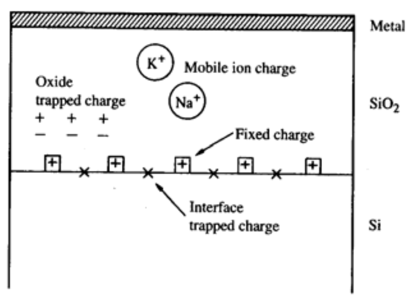
$\propto e^{q\varphi_S/kT}$



$$V_g = V_{fb} + \varphi_S - \frac{Q_S(\varphi_S)}{C_{Ox}}$$

- This equation clearly incorporates the effect of subthreshold current in MOSFETs, unlike the simple V_T equations studied previously (for example, in EE130)

Oxide Charges



- In general, these charges all modify the threshold voltage based on their charge centroid

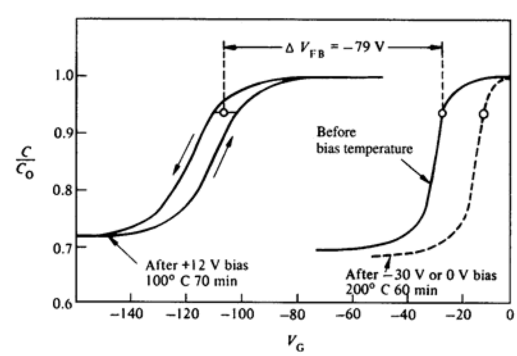
$$\Delta V_T = -\frac{1}{\varepsilon_{SiO_2}\varepsilon_0}\int_0^{x_0} x\rho_{ox}(x)dx$$

- In addition, they may alter mobility due to coulombic scattering

$$V_g = \phi_{MS} + \varphi_S - \frac{Q_S + Q_{it}}{C_{Ox}} - \frac{\int_0^{T_{Ox}} x\rho(x)dx}{\varepsilon_{Ox}} = V_{fb} + \varphi_S - \frac{Q_S}{C_{Ox}}$$

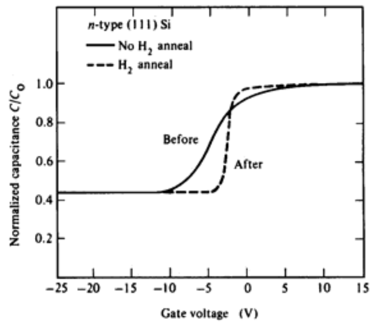
Mobile Ions

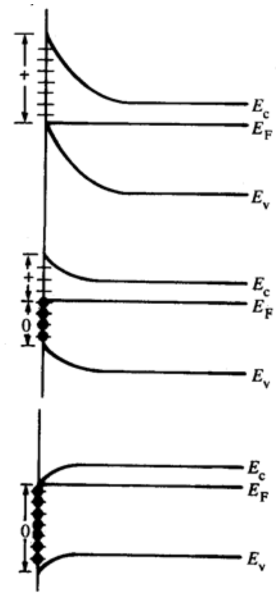
- People observed odd shifts in C-Vs



- Reason: Mobile charge was moving towards / away from interface, changing charge centroid

Interface traps





Traps cause “sloppy” C-V and also greatly degrade mobility in channel

Noise due to Interface Traps

Telegraphic noise in I_d of a small MOSFET is the signature of a signal interface trap.

$$I_d \propto \mu N_{inv} \quad (N_{inv} = n_{inv} \times W \times L)$$

When a single trap changes from “empty” to “filled” $\Delta N_{inv}=-1$.

$$\Delta I_d \propto \mu \, \Delta N_{inv} + N_{inv} \, \Delta \mu$$
$$= -\mu + N_{inv} \, \Delta \mu$$

$$\frac{\Delta I_d}{I_d} = \frac{-1}{N_{inv}} + \frac{\Delta \mu}{\mu}$$

$\Delta \mu<0$: acceptor type
$\Delta \mu>0$: donor type

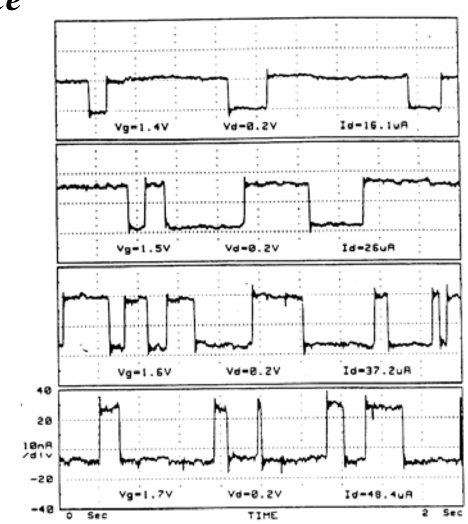
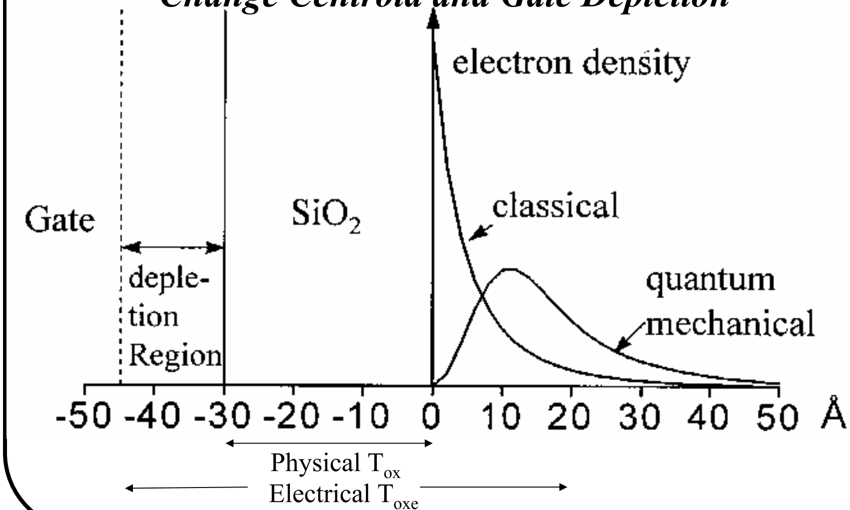
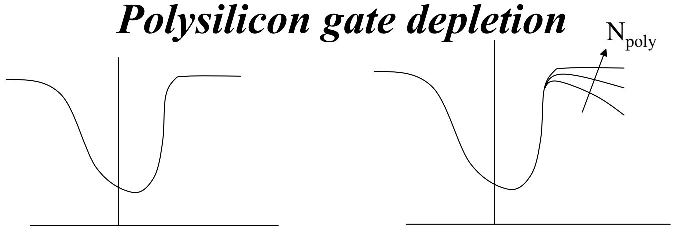


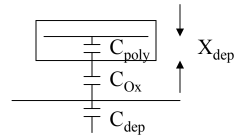
Fig. 2. The fluctuations of drain current at different gate voltages for a small-area n-channel MOSFET (W = 1.2 μm, L = 0.35 μm, T_{ox} = 8.6 nm).

"Electrical Tox" Raised by Inversion Change Centroid and Gate Depletion



Polysilicon gate depletion





Impact on IV

increase the effective Tox by ~5Å -20Å

$$Q_{inv} = \int C dV_g$$

decrease of Vg by ~0.2V,

also has impact on CV and S

$$S = 60mv\left(1 + \frac{C_{dep}}{C_{Ox}}\right)$$

$$C = \left(\frac{1}{C_{ox}} + \frac{1}{C_{poly}}\right)^{-1} = \left(\frac{T_{ox}}{\varepsilon_{ox}} + \frac{W_{dpoly}}{\varepsilon_s}\right)^{-1}$$

$$= \frac{\varepsilon_{ox}}{T_{ox} + W_{dpoly}/3}$$

How to Reduce Gate-Depletion Effect

- **Metal gate** - process integration issues

Or increase active doping concentration in the gate:

- **In-situ or $POCl_3$ doped poly-Si gate**
  - Not suitable for dual-gate CMOS technology
- **Higher dosage for ion-implanted poly-Si gate**
  - Cost, damage and boron penetration issues

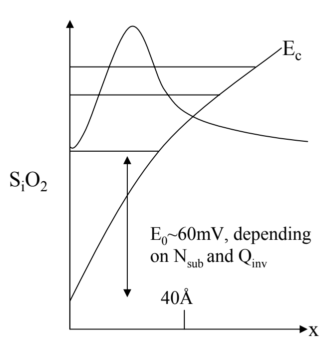
  or higher activation temperature
  - S/D diffusion and boron penetration
- **Poly-$Si_{1-x}Ge_x$-gate technology**

Quantum effect (in the inversion layer)

Sec 3.3 presents the "classical" analysis based on Poisson's equation and Fermi-Dirac function (or Boltzmann's relations)

Quantum confinement in the potential well at the Si/SiO2 interface creates discrete subbands of energy levels.

Ref. Stern, "self-consistent …", Phy, Rev. B. vol. 5, p.4891, 1972



$$E_j \approx \left[\frac{2hqE_S}{4\sqrt{2m_x}}\left(j+\frac{3}{4}\right)\right]^{2/3} \qquad j = 0,1,2$$

$$m_x \approx 0.9m_0 \text{ for } (100)\ Si$$

Assume only ground subband is populated

$$X_{inv} \approx \frac{9\varepsilon_{Si}h^2}{16\pi^2 m_x q\left(Q_{dep}+\frac{1}{3}Q_{inv}\right)}$$

Quantum effect (in the inversion layer)

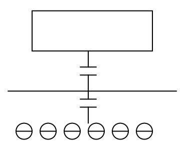
-Effect on V_T

φ_S has to be larger than "$2\varphi_B$"by, say 60mV depending on N_{sub}.

$$\Delta V_t,\ ie\ \ \Delta V_g = \Delta\varphi_S\left(1+\frac{C_{dep}}{C_{Ox}}\right),\quad \sim 100mV$$

Empirical model: Rios, " A Physical Compact MOSFET Model ….", IEDM P.937, 1995

-Effect on CV



$$C_{inv} \equiv \frac{\varepsilon_{Si}}{X_{inv}}$$

-Effect on IV

Similar to CV, but there is a subtle difference between AC charge centroid and DC charge centroid.

"Real" MOSCAP CV Characteristics