UNIVERSITY OF CALIFORNIA
Department of Electrical Engineering and Computer Sciences
CS150 Fall 2001

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Homework #4

Due: Friday, October 12, 2001

1. Find the minimum Sum-of-Products form of the following functions, using K-maps (assume A is the MSB):

   a. f(A,B,C) = Σm(0,1,2,4,5)
   b. f(A,B,C) = Σm(2,3,4,6)
   c. f(A,B,C,D) = Σm(0,6,8,11,14,15)
   d. f(A,B,C,D) = Σm(1,3,5,6,7,8,9,10,11,13,15)
   e. f(A,B,C,D) = Σm(1,2,3,4,10,12) + Σd(8,9,11)
   f. f(A,B,C,D) = Σm(6,7,10,11,12,14,15) + Σd(0,3,13)
   g. f(A,B,C,D,E) = Σm(1,5,6,8,12,24,26,27) + Σd(13,22,28)

Solution

a.

C \ AB	00	01	11	10
0	1	1	0	1
1	1	0	0	1

F = B' + A'C'

b.

C \ AB	00	01	11	10
0	0	1	1	1
1	0	1	0	0

F = A'C' + A'B

c.

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	1	1
10	0	1	1	0

F = B'C'D' + ACD + BCD'

d.

CD \ AB	00	01	11	10
00	0	0	0	1
01	1	1	1	1
11	1	1	1	1
10	0	1	0	1

F = D + AB' + A'BC

e.

CD \ AB	00	01	11	10
00	0	1	1	X
01	1	0	0	X
11	1	0	0	X
10	1	0	0	1

F = BC'D' + B'D + B'C

f.

CD \ AB	00	01	11	10
00	X	0	1	0
01	0	0	X	0
11	X	1	1	1
10	0	1	1	1

F = AB + BC + AD

g.

E = 0

CD \ AB	00	01	11	10
00	0	0	1	1
01	1	1	X	0
11	0	0	0	0
10	0	1	0	0

E = 1

CD \ AB	00	01	11	10
00	0	0	X	1
01	0	X	0	0
11	0	0	0	1
10	0	0	0	1

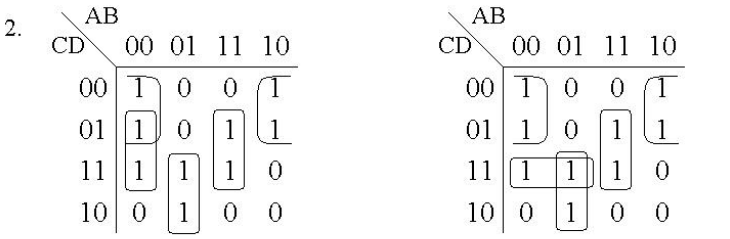
F = AC'D' + A'C'DE' + AB'CE + A'BCD'E'

2. The minimized function is not necessarily unique. Given the following function, find two different expressions that are both minimum:

   f(A,B,C,D) = Σm(0,1,3,6,7,8,9,13,15)

Solution

2.



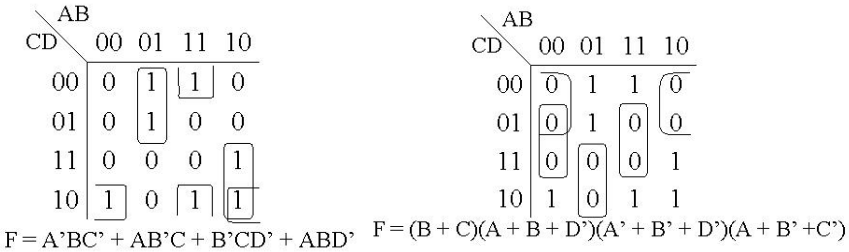
F = B'C' + A'B'D + ABD + A'BC F = B'C' + A'CD + ABD + A'BC

3. Find the minimum Product-of-Sums form of the following function, and compare it to the minimum Sum-of-Products form:

   f(A,B,C,D) = ΠM(0,1,3,6,7,8,9,13,15)

Solution

3.



F = A'BC' + AB'C + B'CD' + ABD' F = (B + C)(A + B + D')(A' + B' + D')(A + B' +C')

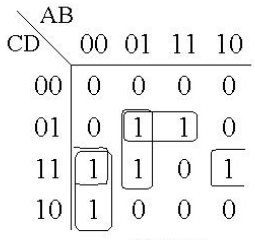
4. Write the truth table of a combinational function whose output P is 1 if the input A[3..0] is the binary representation of a prime number. Find a minimized function to compute P using a K-map.

Solution

4.

Line #	A_3	A_2	A_1	A_0	P
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Truth table



K-maps

P = BC'D + A'BD + B'CD + A'B'C

5. The No-Homers Club of Springfield is directed by a committee consisting of 4 members (P,A,B,C). Some issues require absolute majority, i.e., the vote of at least 3 members. Other issues are more relaxed and can be decided by the vote of the president (P) in case of divided opinion, i.e., if there are 2 favorable votes and one of them is the president, then the decision gets approved.

    a. Write a truth table of the two decision rules (called D1 and D2), assume an input takes the value "1" when the correspondent member votes affirmatively, and the output takes the value "1" when the decision is approved.
    b. Write a minimum expression for the outputs, using K-maps.

Solution

5

P	A	B	C	D1	D2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

Truth table

D1

BC \ PA	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	0	1	1	1
10	0	0	1	0

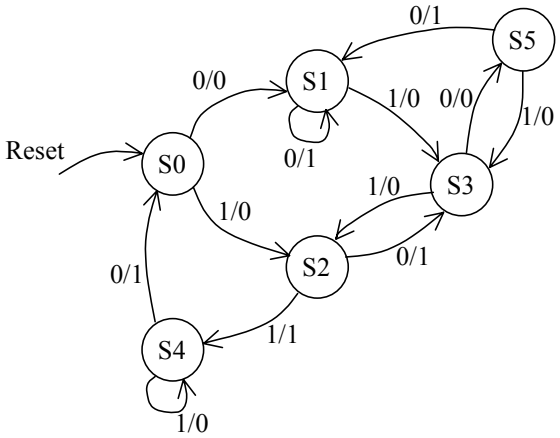
D2

BC \ PA	00	01	11	10
00	0	0	1	0
01	0	0	1	1
11	0	1	1	1
10	0	0	1	1

K-maps

D1 = ABD + ABC + BCD + ACD
D2 = AB +AD + AC + BCD

6. Given the following FSM diagram of a Mealey machine, write the state transition table and find an equivalent FSM with a minimum number of states, using implication charts. Write the new state transition table and the new FSM diagram.



Solution

6.

	In = 0		In = 1	
C.S	Out	N.S	Out	N.S
S0	0	S1	0	S2
S1	1	S1	0	S3
S2	1	S3	1	S4
S3	0	S5	0	S2
S4	1	S0	0	S4
S5	1	S1	0	S3

C.S – current state, N.S. – next state

State transition table

S1	X				
S2	X	X			
S3	S1-S5 S2-S2	X	X		
S4	X	S1-S0 S3-S4	X	X	
S5	X	S1-S1 S3-S3	X	X	S0-S1 S4-S3
	S0	S1	S2	S3	S4

Initial implication chart

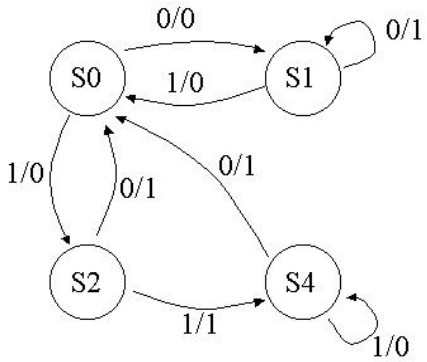
S1	X				
S2	X	X			
S3	S1-S5 S2-S2	X	X		
S4	X	~~S1-S0~~ ~~S3-S4~~	X	X	
S5	X	S1-S1 S3-S3	X	X	~~S0-S1~~ ~~S4-S3~~
	S0	S1	S2	S3	S4

First marking pass

S0 = S3, S1 = S5

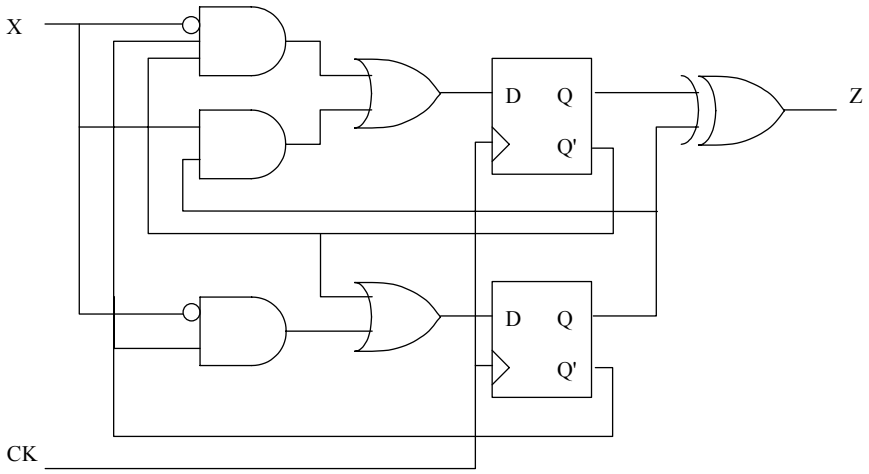
	In = 0		In = 1	
C.S	Out	N.S	Out	N.S
S0	0	S1	0	S2
S1	1	S1	0	S0
S2	1	S0	1	S4
S4	1	S0	0	S4

New state transition table



New FSM

7. Given the following circuit, write the state transition table and find an equivalent FSM with a minimum number of states, using implication charts. Write the new transition table. Do you recognize this well-known device?



Solution

7.

$Q1 = X' Q2' Q1' + X Q2$

$Q2 = X' Q2' + Q1'$

$Z = Q1 \oplus Q2$

S1	X		
S2	X	S1 – S1 S3 – S0	
S3	S3 – S0 S1 – S2	X	X
	S0	S1	S2

Implication chart

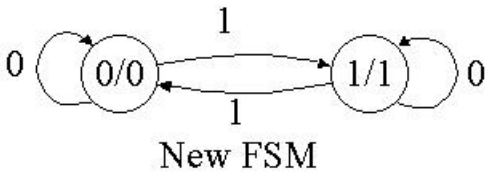
$S0 = S3$, $S1 = S2$

C.S	In	N.S. / Out
Q	X	Q
0	0	0
0	1	1
1	0	1
1	1	0

New state transition table

	C.S.		In		N.S.		Out
	Q1	Q2	X		Q1	Q2	Z
S0	0	0	0	S3	1	1	0
S0	0	0	1	S1	0	1	1
S1	0	1	0	S3	0	1	1
S1	0	1	1	S3	1	1	0
S2	1	0	0	S1	0	1	1
S2	1	0	1	S0	0	0	0
S3	1	1	0	S0	0	0	0
S3	1	1	1	S2	1	0	1

State transition table



New FSM

This device is a parity checker.