UNIVERSITY OF CALIFORNIA
Department of Electrical Engineering and Computer Sciences
CS150 Fall 2001

Prof. Subramanian

Homework #1

Due: Friday, September 14, 2001

1. Which of the following contain circuits that are likely to be combinational and which contain sequential circuits? Explain the rationale behind your answer!
    a. A circuit that takes as an input a number between 0_{10} and 15_{10} and outputs the square of the input.
    b. A circuit that takes two numbers, one at a time, for input and outputs the sum of the two numbers.
    c. A circuit that takes a year (ex. 2001), in decimal form, and outputs a 1 if it is a leap year, 0 if it is not.
    d. A circuit that takes as an input an alternating stream of 1's and 0's and inverts its one output bit after receiving 2 alternations on the input.
    e. A circuit which has eight binary inputs, numbered 0-7, and three binary outputs and works as follows. The output is the binary representation for the lowest numbered input which is a 1. If no inputs are a 1 the output is all zeros. (For example, if the input is 00100111 on inputs 0-7 respectively, the output will be 010_2 or 2_{10}. If the input were 00000011, the output would be 110_2 or 6_{10})

Solution

a. combinational
b. sequential
c. combinational
d. sequential
e. combinational

2. Consider all possible functions of two Boolean input variables.
    a. Write down a truth table that enumerates all of these possible functions.
    b. Write down the Boolean equations for each of these functions.
    c. Which function(s) is/are the most complex, i.e. would take the most gates to realize?
    d. How many possible functions are there of three Boolean variables? Four variables? Twenty?

Solution

a.

X	Y	f0	f1	f2	f3	f4	f5	f6	f7	f8	f9	f10	f11	f12	f13	f14	f15
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

b.

f0 = 0

f1 = X • Y

$f2 = X \bullet Y' = (X'+Y)'$
$f3 = X$
$f4 = X' \bullet Y = (X+Y')'$
$f5 = Y$
$f6 = X' \bullet Y + X \bullet Y' = X \oplus Y$
$f7 = X+Y$
$f8 = X' \bullet Y' = (X+Y)'$
$f9 = X \bullet Y + X' \bullet Y' = (X \oplus Y)'$
$f10 = Y'$
$f11 = X+Y' = (X' \bullet Y)'$
$f12 = X'$
$f13 = X'+Y = (X \bullet Y')'$
$f14 = X'+Y' = (X \bullet Y)'$
$f15 = 1$

c. The functions which take the most gates and transistors to make are the XOR (f6) and XNOR (f9) functions.

d. In general there are 2**(2**n) possible functions of n variables. This means 256 functions of 3 variables, 64K=65,536 functions of 4 variables and 2**1G=2**1,048,576 (a HUGE value) functions of 20 variables.

3. Given the following Boolean equation:

$$f(X,Y,Z) = \big((X+X') \bullet Y + (X \bullet Y \bullet Z') + Z \bullet (X+Y') \bullet (X'+Y) \big) \bullet (X + Y + Z')$$

   a. Write out the truth table.
   b. Can you see a way that this equation could be simplified? If so, what is it?

Solution

a.

X	Y	Z	f(X,Y,Z)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

b.

$$f(X,Y,Z) = \big((X+X') \bullet Y + (X \bullet Y \bullet Z') + Z \bullet (X+Y') \bullet (X'+Y) \big) \bullet (X + Y + Z')$$

$$f(X,Y,Z) = \big(Y + X \bullet Y \bullet Z' + Z \bullet (X \bullet X' + X \bullet Y + X' \bullet Y' + Y \bullet Y') \big) \bullet (X + Y + Z')$$

$$f(X,Y,Z) = \big(Y + X \bullet Y \bullet Z' + X \bullet Y \bullet Z + X' \bullet Y' \bullet Z \big) \bullet (X + Y + Z')$$

$$f(X,Y,Z) = \left(Y + X' \bullet Y' \bullet Z \right) \bullet (X + Y + Z')$$

$$f(X,Y,Z) = \left(Y \bullet (X + Y + Z') + X' \bullet Y' \bullet Z \bullet (X + Y + Z') \right)$$

$$f(X,Y,Z) = \left(Y + X' \bullet Y' \bullet Z \bullet X + X' \bullet Y' \bullet Z \bullet Y + X' \bullet Y' \bullet Z \bullet Z' \right)$$

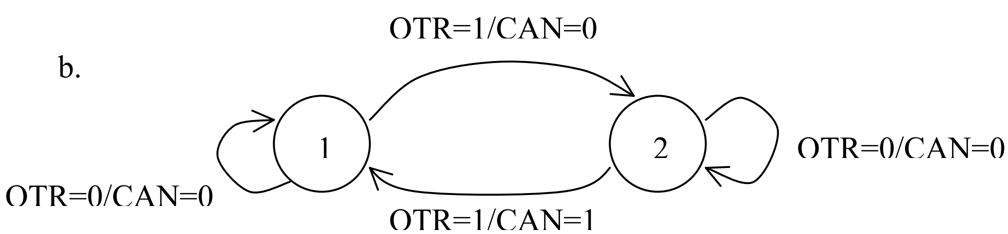
$$f(X,Y,Z) = Y$$

4. A vending machine sells 12 oz cans of Slurm for 50¢. The machine only takes quarters. A digital circuit inside the machine determines how much money has been inserted and when to dispense its product.
   a. Define the system's inputs and outputs
   b. Draw a finite state diagram for this subsystem, showing states, transition arcs, and logical conditions under which the machine moves from one state to the next.

Solution

a. Inputs: QTR - signal indicating that a quarter has been introduced.
   Outputs: CAN - signal indicating that a can should be delivered.

b.



5. Consider if the beverage machine in problem 4 could accept other forms of coinage.
   a. What other inputs and outputs would you need?
   b. What other states would you need?
   c. What if the machine contained products of varying cost? How could the machine make change? What about a coin release button?

Solution

a. I would need one input for each type of coin (quarter, dime, nickel), but I would not need to add more outputs.
b. I would need one state for each possible cumulative value introduced in the machine, i.e., 0c, 5c, 10c, 15c and so on.
c. I would need an additional input (or set of inputs) that would indicate which is the product selected and an additional output (or set of outputs) to indicate which is the product delivered and when. There are many interesting options in the design of the machine: the user should select the product before or after introducing the coins? If there is a coin release button, it would be an additional input which would make the machine return the coins that were introduced, so we would need also an additional output to indicate that.